Expressing Existence

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Chapter One

The Importance of Quality

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I. Quality, Not Quantity, Determines Existential Import

When Kant declared that "Being is obviously not a real predicate" (p. 504) he was in part foreshadowing a post-Fregean view. Kant and followers of Frege agree that existence claims ought to be explicated in such a way that the function of the natural language/grammatical predicate 'exists' is accomplished by a purely logical (non-predicate) part of speech. For Kant, and the Aristotelian tradition he was part of, the copula - an affirmative assertoric form of the verb 'to be' - has this existential force.¹ In the same paragraph Kant explains that "Logically it [being] is merely the copula of a judgment". So

'Brown cows exist'
is treated as

'Cows are brown'

¹ This is part of Kant's treatment of 'exists' in general/formal logic. It underlies his views in what he calls transcendental logic. See Orenstein, Existence and the Particular Quantifier, Chapter 4 and the Appendix: The Copula and Existence.
with 'are' the plural form of the verb 'to be' having existential force. The copula connects, but is not, a traditional subject or predicate term.

This Aristotelian theme was refined in the twentieth century when Lesniewski formulated the axiomatized system he named "Ontology". A singular form of the copula was taken as primitive and deployed to contextually define existence sentences. Following Kotarbinski (p. 196), I use the Latin 'est' to represent this copula. Ontology is a calculus of nouns - singular, common, and empty. With the bold letters $a, b$ serving as noun variables the definition is

$$b \text{ exists iff } (\exists a)(a \text{ is a/est } b).$$

So,

$$\text{Cows exist iff Some thing (e.g., Bossie) is a/est cow.}$$

(Ajdukiewicz (p. 210)

Most current philosophers, however, understand "existence is not a predicate" differently. This more recent way of understanding the remark originated in Frege's development of modern logic; it was adopted by Russell and others in the Logical Empiricist as well as the Anglo-American tradition, and forms the basis for Quine's account of ontological commitment. Like Kant and Lesniewski, they too maintain that existence sentences can be explicated in terms of sentences with a non-predicate logical concept having existential force.

So

'Brown cows exist'

is treated here as

'Some cows are brown'.

In this tradition it is the quantifier 'Some' that has existential force. Quine summed up the matter by remarking that "Existence is what existential quantification expresses" (1969, p.97) Stated in the language of modern predicate logic the sign '(\exists x)' has this existential role. The sign is interpreted and read in English as 'For some x', 'There is at least one x' or 'There exists an x'. Since Frege this quantifier has been referred to as "the Existential Quantifier" and

'Brown cows exist'

appears in quasi-predicate logic notation as

'(\exists x)(x \text{ is a cow and } x \text{ is brown})'.

Natural language existence claims are restated along these lines and whenever this quantifier governs a formula there is an existence claim. Quine has made two illustrative remarks about this stance. He tells us that "Existence is what existential quantification expresses" and thereby makes the connection between existence and '(\exists x)' appear to be guaranteed as a matter of mere nomenclature. (Quine, 1969, p.97) Better known is his play on Bishop Berkeley's remark. Berkeley told us that to be is to be perceived. Quine said "to be is to be the value of a variable". (Quine, 1948, pp.13-5) As he sees it, in order for existential generalizations to be true, the variable in the open sentence
'x is a cow and x is brown',
bound by the quantifier must have at least one object/one value
satisfying that open sentence. Sentences containing the variable,
which the quantifier governs, are true when there are values (objects)
that, so to speak, satisfy the variables.

The Frege through Quine view is the dominant one and it
receives much of its support from its connection with modern
predicate logic. In what follows, I argue for a variant of the older
Aristotelian tradition as a rival to the Frege inspired quantifier view. I
offer a way of thinking about existence claims in modern predicate
logic by calling attention to notions that are already in predicate logic
and which were recognized in the older Aristotelian tradition. Of
particular importance is one that warrants being called an affirmative
- negative distinction. It is grounded in contrasting singular sentences of
a certain sort with their external negations. Singular sentences play a
role in determining existential import and ontological commitment.
Conceived of in this way, affirmative sentences provide an alternative
model to that currently accorded to generalizations, and, in particular,
to so-called "existential" generalizations. Employing singular
sentences in this fashion has precedents in the history of logic and
metaphysics.

This work aspires to be part of the tradition in modern logic and
its philosophy, that takes cognizance of the history of logic and tries
to discover, and, at times, to absorb themes from the old logic into the
new one. It

contrives both to use modern techniques to bring out more
clearly what the ancients were driving at, and to learn from the
ancients useful logical devices which the moderns have in general
forgotten. (Prior, 1952, p.37, Copeland, p. 5)

The terms 'quality' and 'quantity' of traditional logical theory
were used to classify what we now call truth vehicles i.e., sentences,
propositions, and judgments. They are classified according to quality
mainly in virtue of their being affirmative or negative. 'Quantity'
pertains primarily to whether a judgment is universal, particular
("existential" in current parlance) or singular. I will construe 'quantity'
as referring solely to generalizations, that is, to quantifications.
Singular sentences will be dealt with under the quality heading as
affirmative. Doing this proves to have a certain utility for making the
affirmative - negative distinction underlying an older conception of
existential import. This essay is not primarily intended as a work in the
history of logic. Older themes are stated independent of their
historical context and justified on their own in neutral terms of current
logical theory.
These older insights ought to be recognized in current work because they are warranted and not merely because of their importance for the history of the subject. This chapter attempts to provide an overview of the differences between a quantifier paradigm and an affirmative-negative paradigm. Since it is an overview several issues will be passed over. They will be taken up later, e.g., in an appendix to this chapter on Frege and Ayer's criticisms of the concept of quality, in later chapters dealing with Russellian, object dependent and Free logic treatments of vacuous/empty terms, and in sections on truth conditions. To take up such issues here would cause the chapter to grow to an unmanageable length. Worse still, it will interfere with providing an overall perspective.

The arguments provided for the quality paradigm are entirely in terms of predicate logic. Whatever doubts Frege, Ayer and others have had about the distinction, standard approaches to the syntax and semantics (truth conditions) for predicate logic allow one to distinguish between certain well formed formulas containing an element of negation and those that do not. The affirmative-negative distinction I have in mind is quite like the distinction between atomic sentences and their negations found in predicate logic. In fact, the expression 'literals' is employed at times to refer to precisely these atomic sentences and their negations, e.g., Raymond Smullyan uses the set of literals to formulate a version of Henkin's completeness proof for first order predicate logic. I am proposing that certain basic "affirmative" sentences, such as atomic sentences, have existential import, and that any sentences which require their truth inherit that existential import. By contrast, sentences, such as negations of atomic sentences, whose truth does not require the truth of such basic sentences, do not have existential import. The slogan of Aristotelian logic that quality not quantity determines existential import is a plausible one (Thompson, p.60). However, it must be understood in terms of singular-atomic sentences - the "primitive" or "basic grounding" affirmative sentences - and their negations. This is the affirmative - negative distinction that furnishes the crucial distinction of quality.

The full statement of the distinction of quality is based on the distinction between singular sentences and their external negations. The notion of a singular sentence is broader than that of an atomic sentence. Atomic sentences can be thought of, roughly speaking, as a variety of singular sentences, as those containing no complex parts. Non-atomic singular sentences can contain complex predicates such as internal negations, e.g. 'Alcibiades is unwise'. Singular belief sentences constitute another non-atomic case, e.g., 'Tom believes that it is snowing' (where 'believes that it is snowing' is a complex predicate). What all singular sentences have in common is that the predicate, whether atomic or not, is treated in a truth condition in the same way as an atomic predicate is. In both atomic and non-atomic cases truth is
a matter of the predicate (simple or complex) applying to the referent of the singular term, i.e., the singular term’s referent is one of the objects to which the predicate applies. Where it makes no difference to the discussion, I will use 'singular sentence' and the special case 'atomic sentence' interchangeably. The full and accurate account though is of singular sentences and their external negations. For purposes of exposition, in this first chapter I will try to appeal to singular sentences when they take the form of atomic sentences.

A plausible view of the truth conditions for atomic sentences and their negations accords existential commitment to atomic sentences but not to their negations. Consider how an atomic sentence could be said to be true. As a start one might somewhat figuratively say that the sentence corresponds to reality, or that things are as the sentence describes them, or (a sixty's version) "tells it like it is". So, if true, the sentence has existential import and ontologically commits us to whatever objects are required for its truth. Where there are no objects to correspond to the sentence, it can plausibly be said to be false since there is no reality to which it corresponds. The negation of such a sentence would be true. Such a negation does not have existential import since it does not require the existence of the relevant (or any other) objects. The above figurative remarks concerning correspondence, etc., can with generosity be construed as metaphorical versions of the more careful accounts found in standard truth conditions such as those given in chapter two. The sentences 'Socrates is human' and 'This shirt is made of cotton' are true when the subject's referent, its singular denotation, is one of the objects to which the predicate applies (Ockham, p. 86). A model/set theoretic variant says that the sentence is true when the semantic value (the designation) of the subject, 'Socrates', is a member of the set (the extension) that is the semantic value of the predicate, 'is human'. Another variant is that the individual the subject term denotes has the property the predicate expresses, i.e., Socrates has the property of being human. These three approaches may well differ in their ontological assumptions but for the purposes of this introduction any one of them will do. On each of these approaches an atomic sentence ‘Fa’, e.g., 'Socrates is human', requires for its truth that a (Socrates) exists and that F's (humans) exist.

But now consider four ways in which such singular-atomic sentences can fail to be true (given bivalence they are false), and their negations '¬Fa' are true:

1. Socrates ate pepperoni pizza.
2. Socrates is a flying horse.
3. Pegasus ate pepperoni pizza.
4. Pegasus is a flying horse.
1 is false because though Socrates exists tenselessly and pepperoni pizza consumers do, too, Socrates is/was not one of them. So the negation of 1, \( \neg (\text{Socrates ate pepperoni pizza}) \), is true.

2 is false because there are no and never have been any flying horses, so Socrates is not one of them. Thus, the negation of 2, \( \neg (\text{Socrates is a flying horse}) \), is true.

3 is false because though pepperoni pizza consumers exist, Pegasus does not, so he cannot be one of them. The negation of 3, \( \neg (\text{Pegasus ate pepperoni pizza}) \), is true. We will postpone questions about names and vacuous ones to chapter two where they will be taken up in more detail.

An important ingredient of this work is taking seriously the maxim of conservatism. All other things being equal, when choosing between theories, decide in favor of the one that requires the fewest revisions of the background assumptions. Applied to theories that assign logical forms to natural language sentences, we are cautioned to be conservative in revising natural language intuitions. It clearly prohibits taking too high-handed an approach to sentences with vacuous/empty singular terms and declaring them meaningless, or as not expressing propositions or of not having a truth value. In later chapters there will be further discussion of these points.

4 is false for either of the reasons the sentences in 2 and 3 are false. The negation of 4, \( \neg (\text{Pegasus is a flying horse}) \), is true. (Please put aside the role such sentences play in fiction.)

On this view a sufficient condition (but not a necessary one), given bivalence, for the falsity of an atomic sentence is the vacuity/emptiness of its subject or its predicate terms, i.e., the non-existence of these expressions' purported referents. This also suffices for the truth of its negation. Since such negations can be true because of the vacuity of their referential parts, they have no existential import. They involve no ontological commitment and the same holds for sentences that are logical consequences of them. This provides the basis for Thompson's maxim that quality (the affirmative/negative distinction) determines existential import and not quantity (the universal/particular distinction). In the present work, the expression

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2 This differs from the treatment of singular sentences in Aristotle and the Terminists where such sentences are dealt with in a tensed fashion. In this respect I am more in keeping with the Lesniewskian approach to singular sentences.

3 If one relinquishes bivalence, matters are not significantly changed, since an affirmative atomic sentence's truth suffices for existential import and with the absence of such a truth we have no such import. With bivalence, the absence means that negations of atomic sentences have no existential import. Without bivalence, whatever does not require a true atomic sentence, whether it is false, a third value, or neither true nor false, has no existential import.

4 I am obviously indebted to Manley Thompson's work. However, his use of this maxim differs from that in this paper. Here, quality, the affirmative/negative
‘singular sentence’ applies only to affirmative sentences. They are of the form \( Fa \). The expression ‘affirmative singular sentence’ is redundant. Negations of such sentences (unless otherwise indicated) are sentences of the form \( \neg Fa \). These are negations of singular sentences. They are not singular sentence though they have singular sentences as components. ‘It is not the case that Pegasus is a flying horse’ is not a singular sentence.

This quality paradigm differs from the entrenched Frege through Quine view that quantity - the so-called "existential quantifier" - determines existential import. The quality approach fits in with the insight that affirmative singular sentences with vacuous singular terms or vacuous predicates are false (there is no reality to which they correspond) and that accepting the negations of such false singular sentences does not ontologically commit us to anything. As we shall see, the quantifier approach does not conform to these insights.

II Quality: What Existential Import Is

The thesis that quality and not quantity determines existential import contains a positive and a negative claim. The positive one is that quality (the affirmative/negative distinction as per atomic-singular sentences, i.e., atomic-singular sentences and their negations) determines existential import. The negative claim is that quantity (whether a generalization is a universal quantification or a so-called "Existential" quantification), does not determine existential import. In the Polish tradition '\((\exists x)\)' was called the small quantifier and '(x)' the large one. For want of a better term, I will speak of the particular or the so-called "existential" quantifier, or, even somewhat uncomfortably go along with the practice of referring to it as the existential quantifier.

The positive thesis asserts that a given sentence has existential import if and only if the truth of an atomic sentence is a necessary condition for that given sentence's truth. Existential import is a matter of true predications in affirmative atomic sentences.

It is evident from the different versions of the truth condition for atomic sentences that neither the singular subject nor the predicate can be vacuous when the sentence is true. This non-vacuity is the source of existential import. Any sentence that requires for its truth the truth of an atomic sentence inherits the ontological commitment/existential import of that atomic sentence. Those sentences that "depend" on negations of atomic sentences, in the sense that the truth of such a negation suffices for the truth of the

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distinction, is tied to atomic sentences and their negations, whereas Thompson ties it to the four standard categorical sentences.
sentence in question, do not have existential import. Since the negation of an atomic sentence does not have existential import, any sentence following from such a negation can be true without requiring the existence of anything.

A sentence, S, has existential import if and only if there is an atomic sentence, A, whose truth is required for S's truth (an atomic sentence is necessary for S's truth):

S has existential import ↔ (S is true requires that A is true), where A is an atomic sentence

S, does not have existential import if a negation of an atomic sentence, ¬A, suffices for S's truth (if the negation of an atomic sentence implies S):

¬ (S has existential import) → (¬A implies S)

III. Quantity: What Existential Import-Ontological Commitment Is Not

The negative claim of the thesis is that quantity does not determine existential import. It opposes the entrenched view of existential import and ontological commitment initially stated by Frege, and adopted by Russell, Quine, and many others. According to this widely accepted view, the distinction of quantity, a sentence being "existential"/particular, determines existential import: "Existence is what existential quantification expresses" (Quine, 1969, p. 97).

The term 'existential' in 'existential quantifier' begs the question for the view that the quantifier has a privileged connection with existence. It gives Quine's aphorism the appearance of expressing a tautology. On the Frege-Russell-Quine held view, every sentence governed by the some/particular quantifier (or implying such sentences) has existential import. Despite its popularity, this view is not without its problems. Later we shall consider a family of puzzles/paradoxes involving the above so-called "existential" quantifier. Some start by taking as a premise a negation of a false vacuous singular sentence (affirmative), which, as such, has no existential force. We then derive a particular generalization from this negation by the rule of particular generalization (so-called "existential" generalization). When the conclusion is construed as having existential import, as it is in the Frege-Russell-Quine tradition, there is a paradox. How can a conclusion asserting existence follow from a premise whose truth does not require existence?

This type of argument takes a particularly well focused form in the familiar Plato's Beard puzzle (considered in the second and third chapters) and in the problem of the empty domain (taken up in
chapter four). The premise of the Plato's beard puzzle-argument is a singular denial of existence stating that Pegasus does not exist. The conclusion that follows by the rule of existential generalization says that something does not exist. On the view proposed in this work the puzzle is presented as an example of how a true premise without existential import yields a purported existential conclusion when the 'some' quantifier is construed as having existential force. Moreover, the conclusion is taken as tantamount to claiming that there exists something that does not exist. Our dissolution of the problem is to argue that the argument is sound; the premise and the conclusion (Something does not exist) are contingent truths with the conclusion following by a valid rule. All of this is explained by providing a variant of standard truth conditions. The dissolution is part of the tradition of dissolving a philosophical problem by indicating how it involves a misuse of ordinary-natural language (regimenting 'something' as 'there exists'). But there is also an element of a solution in that a theoretical framework is supplied, i.e., truth conditions are given which reflect the use of the ordinary-natural language quantifier.

IV. Every Logical Constant is What It Is and not Another Thing

In order to do justice to the relation of quality to quantity, i.e., of singular sentences (affirmative) as vehicles of existential import to the 'all' and 'some' quantifiers we take two steps. We provide: 1. a material adequacy condition for accounts of 'all' and 'some' and 2. referential truth conditions.

Material adequacy conditions provide constraints on giving an account of a concept. The constraints insure that essential features of the concept are captured by the theory or definition offered of it. They provide a "criterion" or standard for judging how adequate the definition or theory is to the central intuitions embodied in the concept. Tarski's famous convention T is just such a condition. Sentences of the form: 'Snow is white' is true if and only if Snow is white, should be consequences of the definition. Tarski intended it to guarantee that intuitive features of an Aristotelian realist/correspondence style conception of truth were captured. In the Tarski case, sentences having a form dictated by convention T must follow from the definition of truth. The use of a material adequacy condition for 'all' and 'some' in the present work is less stringent. We shall only require that there be no cases in the accounts of the quantifiers that violate the adequacy condition.

The adequacy condition for 'all' and 'some' should come as no surprise. It is that 'all' is "strictly and formally analogous" to 'and' and
'some' to 'or'. 'Everything is in space' is so closely related to 'a is in space and b is in space and etc.' that an account of the generalization that did not preserve its close relation to the conjunction, would fail to account for the universal claim. Similar remarks apply to the relation between 'Something is blue' and 'a is blue or b is blue or etc.' "Universality" is intrinsically linked with conjunction and "at least oneness" with disjunction. What I mean by "strictly and formally analogous" is that the logical operations of universal and particular quantification must parallel the logical relations found in their conjunctive and disjunctive counterparts. The logical principles (rules or theorems) which apply to the quantifiers should be analogous to principles applying to conjunctions and disjunctions. Three kinds of quantificational and sentence logic principles play a central role. Universal instantiation is an analogue of conjunctive simplification ('and' elimination). Particular-"Existential" generalization is analogous to disjunctive addition ('or' introduction). There are parallel duality relations in predicate and in sentence logic. In predicate logic they are known as "quantifier interchange" or defining one quantifier in terms of another. Their analogues in sentence logic are known as "De Morgan's laws".

The analogies for universal instantiation and "Existential" generalization are:

\[
\begin{align*}
\text{p & q} & \quad \text{(...a....) & (...b...) & etc.} & \quad \text{(x)(.....x...)}
\text{p} & \quad \text{(...a....)} & \quad \text{(..a....)}
\text{p v q} & \quad \text{(...a....) v (...b...) v etc.} & \quad \text{(∃x)(.....x...)}
\end{align*}
\]

One of the analogous instances of Duality is:

\[
\begin{align*}
\text{p & q} & \quad \text{(...a....) & (...b...) & etc.} & \quad \text{(x)(.....x...)}
\text{\neg(\neg p v \neg q)} & \quad \text{\neg(\neg(...a....) v \neg(...b...) v etc.)} & \quad \neg(\exists x) \neg(...x...)
\end{align*}
\]

To have an authentic account of 'all' and 'some' the principles (rules of inference, logical truths, etc.) governing their use must accord with the analogous sentence logic principles. Universal instantiation is like conjunctive simplification and "Existential" generalization is similar to disjunctive addition. The various forms of quantifier interchange are extensions of De Morgan's laws.

In addition to being highly intuitive there are several ways in which a similar role for the analogies is recognized in the literature.
1. In first order predicate logic with one-placed predicates this relationship of the quantifiers to the connectives appears in the form of a decision procedure for monadic predicate logic (Behmann). By suitably expanding a one-placed predicate logic formula into conjunctions and disjunctions by replacing universal (particular) generalizations with suitable conjunctions (disjunctions) of their instances we can determine in truth functional logic the truth assignments to the original quantifications and thereby test for logical truth, implication, etc.

2. Expansions appear in Terminist logic as the doctrine known as "descent to the singular". In the theory of supposition of the Terminist tradition of Ockham, Buridan, and others, generalizations are explained, some even say, given truth conditions, in terms of a doctrine of descent to singulars. This involves restating categorical sentences in terms of conjunctions and disjunctions of their universal and particular features. Numerous authors, e.g., Thompson, Patterson and others, make use of the analogies to establish results about generalizations.

3. Some of the more reputable contemporary introductory texts explain the natural deduction rules for the quantifiers in terms of the analogous truth functional connectives, e.g. Lemmon. Kalish and Montague adopt a notation for quantifiers as visually making the connection apparent, i.e., the lower case 'v' is a sign for disjunction and a larger 'V' is used as part of the sign for an "existential" quantifier. The signs for conjunction and the universal quantifier are upside down versions of 'v' and 'V'. A further virtue of this notation is that it hints at the aspect of duality inherent in these notions.

In taking it as a material adequacy condition that universal generalizations are strictly analogous to conjunctions and "existential" generalizations to disjunctions, I am offering a more modest claim than that of defining these quantifiers in terms of conjunction and disjunction. (Orenstein, 1990, pp.246-47) Many readers will accept this material adequacy condition without requiring further explanation. For those who have trouble with seeing the modesty of the claim one might invoke the notion of "supervenience" to gesture at what I have in mind. I am not claiming to have given a conceptual analysis of 'all' in terms of 'and', nor of reducing universality to conjunction, nor of having given truth conditions for the universal quantifier in terms of conjunctions. My material adequacy condition is more like holding that universality supervenes on conjunction and particularity, i.e., at least oneness, supervenes on disjunction. An even weaker way of viewing these all/some - and/or analogies, which I offer those still skeptical, is to regard the analogies as a heuristic device that can be discarded and criticism directed to the results the analogies indicate. For those who still need further justification appendix B might help.

In chapter two, referential semantics / truth conditions will be provided. It is a variant of the method Benson Mates gave in his
Elementary Logic. These truth conditions are referential and will be compared with Tarskian and with substitutional approaches. The Mates-like conditions will incorporate the features of singular/atomic sentences mentioned above. The quantifiers will be treated in a referential way and in keeping with the adequacy condition just outlined. As formulated by Mates these conditions have made their way into numerous texts. And while I cite Mates as the source of these truth conditions, Gareth Evans (1996, p. 83) follows Dummett (1973, chapter 2 and pp. 516-17) in claiming that Frege employed just such conditions.

Two types of argument will be appealed to. One is the use of logical principles such as universal instantiation and "existential" generalization and the accompanying corresponding Mates-like truth conditions. The other employs the all/some - and/or analogies. Since the issues bear on existential import and ontological commitment, a number of these arguments concern sentences with vacuous terms. The treatment proposed is offered as a logic free of existence assumptions. It is not a "free logic" as that term is narrowly used by some of its proponents. They restrict themselves to systems in which the particular quantifier is the means for expressing existence claims. In chapters two, three and four, criticisms will be offered of such "free logics". For example, it is common in such restrictive systems of free logic to revise the standard particular generalization rule. Instead of the standard rule we are offered a revised one.

The standard version of "existential" generalization meets our adequacy condition.

\[(\ldots a\ldots) \quad \text{therefore} \quad (\exists x)(\ldots x\ldots)\,.
\]

It is an analogue of

\[(\ldots a\ldots) \quad \text{therefore} \quad (\ldots a\ldots) \lor (\ldots b\ldots) \lor \text{etc.},
\]

The revised rule these free logicians offer is

\[(\ldots a\ldots), \text{ a exists} \quad \text{therefore} \quad (\exists x)(\ldots x\ldots)\,.
\]

It is not an analogue of

\[(\ldots a\ldots) \quad \text{therefore} \quad (\ldots a\ldots) \lor (\ldots b\ldots) \lor \text{etc.}
\]

Such a revision violates the material adequacy condition. Moreover it is unconservative in the sense that classically acceptable standard rules are rejected/mutilated. Immediate inferences hitherto considered valid are rejected. Thus reasoning from an instance to a particular generalization, e.g., 'That is red, so something is red' is judged invalid given the revised rule. Such revisions might be justified, if there were no alternatives for dealing with vacuous names. One of the goals of this work is to provide a non-revisionary framework for doing this. (See Chapters 2, 3, and 4). It is one thing to offer a theory that allows for the presence of vacuous singular terms, but quite another to rule out
hitherto acceptable inferences. The goal is a logical theory free of existence assumptions without sacrificing classically accepted principles.

V. A Survey of the Remaining Chapters

In the following chapters the prominent role of the concept of quantification in posing and dealing with various problems is contrasted with solutions provided along the lines of the quality framework.

Chapter Two: Plato's Beard - Objects that Don't Exist

'Plato's beard' is the name Quine (1948, pp.1-2) gave to problems surrounding arguments such as

Vulcan/Pegasus does not exist,
therefore, something does not exist.

Using an existentially construed particular quantifier the premise which denies the existence of Vulcan, i.e., \( \neg (\text{Vulcan exists}) \) seems to have the form \( \neg (\exists x)(x = \text{Vulcan}) \). The conclusion has the form of an "existential" generalization: Something does not exist, i.e., \( (\exists x)(\neg x \text{ exists}) \), \( (\exists y)\neg (\exists x)(x = y) \). It states that a "non-being exists". On the existential reading of the quantifier the conclusion is interpreted as saying that there exist things that don't exist. As Quine indicated in *Mathematical Logic* (p.50) this is "a contradiction in terms" and as such false though apparently following by a truth-preserving rule from a true premise. Various treatments of this puzzle will be considered: those of Frege, Russell, Evans-Wiggins, and Strawson are challenged for failing to come to terms with the role actually played in reasoning by singular existentials. Vacuous singular terms yield contingently true denials of existence. Arguments are given against the logical forms provided by Meinong, prevalent forms of Free Logic, and Quine. Particular attention is given to Quine's program of "Quinizing a name and Russelling away the description".

By disassociating existence claims and quantification, the conclusion which is prefixed by a particular quantifier is no longer a contradiction in terms. This goes part of the way to justify maintaining that the argument

'Vulcan/Pegasus doesn't exist, so something doesn't exist'
is sound. The argument consists of a premise and a conclusion each of which is contingently true, and relies on an unproblematic application of the standard particular generalization rule. The premise has no existential force (such a denial of non-being has no existential import) and the particular generalization - the conclusion - has no existential force (following from a sentence without existential import).

Chapter Three: Plato's Beard - Objects that Might Not Have Existed

We return to the puzzle but this time as involving questions about identity representing singular existentials such as 'Pegasus exists' is as '(∃y)(y = Pegasus)'. The Plato's beard argument now takes the form:

\[
\neg (\exists x)(x = \text{Pegasus/Vulcan}) \\
(\exists x) \neg (\exists y)(y = x)
\]

The conclusion is incompatible with the claim that everything exists, \((x)(\exists y)(y = x)\). This formula is a theorem in most systems of predicate logic. But when we allow names apparently contingent truths such as that Julius Caesar existed also are theorems and as such seem to count as being necessary truths.

In his paper 'The Kant-Frege-Russell view of Existence: toward the rehabilitation of the second level view' David Wiggins uses this type of reasoning and in effect extends Quine's version of the problem to non-empty names. Quine's version was for objects that do not exist. Wiggins' is for objects such as Julius Caesar that exist but which need not have done so. Theorists such as Wiggins hold the view that sentences with empty names (as in the first version of the puzzle) do not provide truth vehicles/propositions and that the problem does not arise for empty names. However, the problem reappears in form involving identity and non-empty names. The chapter includes critical discussions of Wiggins' account and of a mainstream conception of "free logic." A solution is then provided within modern predicate logic that has its roots in an older tradition of Aristotelian logic.

Chapter Four: The Empty Domain

On the quantifier paradigm, sentences governed by the some/particular quantifier have existential import and express existence claims. Questions arise concerning "existential" quantifications, such as \((\exists x)(Fx \lor \neg Fx)\) which appear as theorems of logic, and so are said to be necessarily true. In the Introduction to Mathematical Philosophy Russell put the matter as follows:
The primitive propositions in *Principia Mathematica* are such as to allow the inference that at least one individual exists. But I now view this as a defect in logical purity (p. 203).

He noted that such purely logical demonstrations of existence had the same dubious character associated with the Ontological Argument.

The solution outlined by Russell was widely adopted thereafter. In the case of the empty domain (where nothing exists) regard all existential sentences (all formulas governed by the "existential"-particular quantifier) as false. Among these are ones like \((\exists x) (Fx \lor \neg Fx)\) whose subformula \((Fx \lor \neg Fx)\) have the form of a logical truth (a tautology of truth functional sentence logic). Some have proposed versions of Free Logics in which the ordinary rules of inference are revised so that such existential sentences are not derivable as theorems.

Several difficulties arise on the above views. Consider one of them. How can the sentence \((\exists x) (x \text{ is a unicorn } \lor \neg x \text{ is a unicorn})\) which is true in every non-empty domain, even though 'is a unicorn' is vacuous, be transformed into a falsehood in the empty domain? Since the predicate was empty in the otherwise populated domains, there is no relevant difference between considering the sentence for those non-empty domains and the empty one.

On the quality paradigm, the 'Ex' quantifier is not construed as having existential force. Instead, it is analogous to a disjunctive expansion. A sentence of the form \((\exists x) (Fx \lor \neg Fx)\) has as its sentence logic expansion \((Fa \lor \neg Fa) \lor (Fb \lor \neg Fb) \lor \ldots\) The expansion is true and so is the particular generalization. The generalization has no existential import since it does not require a basic affirmative sentence for its truth. To be true in the empty domain, the truth of \('\neg Fa'\) (a negation of a basic affirmative sentence 'Fa') suffices. This is insured since both 'a' and 'F' are vacuous/empty in the case of the empty domain. In the case presented above \('(a \text{ is a unicorn } \lor \neg a \text{ is a unicorn}) \lor \ldots'\) is equally true for empty and non-empty domains. There is no need to modify the usual rules of inference. An ordinary form of classical predicate logic already is an Inclusive - Universally free logic.

Chapter Five: Restricted Quantifiers

Categorical sentences are paradigm cases of natural language generalizations. They also form the subject matter of traditional Aristotelian formal logic. As cases of natural language, they are under discussion in current philosophy, linguistics and logic, e.g., Evans, Wiggins, Davies, Westertahl, Neale, Bach, etc. These authors use binary or restricted quantifiers of a special sort and assign categorical
sentences new logical forms. Part of their motivation is to capture the surface grammar of natural language quantifiers. This can be stated as a constraint. The only difference in logical form assigned to the A form categorical sentence: 'Every whale is ferocious', to the E form: 'No whale is ferocious', to the I form: 'At least one whale is ferocious' and to plural quantifier cases such as 'Few whales are ferocious' should be in the initial quantifier phrases.

Utilizing insights suggested by the all/some - and/or analogies of chapter one yields logical forms that enrich standard predicate logic and meet the above constraint for dealing with natural language generalizations including plural quantifiers.

A bonus accrues as a consequence of this approach: the main theses of Aristotle's and traditional syllogistic logic, including that 'Every A is a B' logically implies 'At least one A is a B', are derivable. Semantic tableaux (as well as natural deduction rules) are provided for restricted quantifiers in keeping with those of ordinary predicate logic. There is an explication in modern logic of several hitherto elusive features of traditional logic, such as the rules of quantity and quality, distribution of terms, Aristotelian modal distinctions, etc.

Chapter Six: Intensional Contexts

The role of quantifiers in discussions of modal and doxastic contexts is yet another case in point of the influence of the quantifier approach. The de dicto - de re distinction is currently treated as one pertaining to the scope of quantifiers with respect to modal and doxastic expressions. When the quantifier occurs solely inside the scope of the modal or belief expression, we are said to have a de dicto context. When the quantifier occurs outside the scope of the modal or belief expression and binds a variable inside the scope of the modal or belief expression, we are said to have a de re context.

The chapter begins by dealing with belief ascriptions. A functor approach is provided for 'believes that' instead of the relational predicate 'believes'. Appealing to the all/some - and/or analogies and to the Mates-like truth conditions it is argued that:

1. The current way of singling out de re cases in terms of quantifiers should be abandoned. The distinction should be made in terms of singular belief ascriptions with the singular term occurring outside as well as inside the scope of the belief functor. Doing so reveals that some purportedly de re cases are really de dicto. These "spurious de re" cases are contrasted with genuine de re ones. By showing how some cases of quantifying in are merely de dicto we can accommodate additional inferences involving belief ascriptions.
In the second part of the chapter two topics concerning modal contexts are considered. We employ the distinction between spurious and genuine de re cases to deal with:
1. Some of Quine's criticisms of modal notions.
2. Controversies surrounding the Barcan Formula - criticisms (Kripke) and defenses (Terry Parson, Timothy Williamson).

In the remaining chapters it will be argued that the quality paradigm provides a superior rival to the entrenched quantifier approach in all the areas outlined above. There are in the main two ways in which this superiority is achieved. The quality model offers a treatment that is more in accord with natural language than the quantifier one. The quality approach provides a unified approach. A unified theory for dealing with problems concerning empty names, the empty domain, empty predicates in the traditional syllogistic, and intensional contexts is not readily available (if it is available at all) on the quantifier approach. By way of achieving these results, I provide insight into and thereby a way of reviving an aspect of a distinguished Aristotelian tradition. Stating these purported advantages here at the outset is tantamount to boasting. The remaining chapters bear the burden of proof.
Appendix A: Frege and Ayer on Quality

Frege (1989, pp. 380-1) and Ayer (1954, pp. 36-65) discussed and criticized a different affirmative-negative distinction than the one being proposed here and claimed that it played no useful role in logic or its philosophy. Ayer offered two types of arguments as to why an affirmative-negative distinction is uninteresting. The first of these arguments does not apply to the present distinction. The second poses a problem for any system of predicate logic, and is solved by acknowledging a scope distinction between internal and external negations. The target of these arguments is the distinctions of quality of traditional logic viewed in the following way. A sentence (presumably either in a natural language or in the notation of a logical language such as predicate logic) is characterized as affirmative or negative depending on the absence or presence in the sentence of any one of a list of previously designated negative expressions. There are a stock of negative expressions in English (e.g., no, none, not, never, un, etc.) or in predicate logic notation (the sentence forming functor of negation). A sentence containing any such negative expressions (or a proposition expressed by such a sentence) is considered negative. A sentence containing none of them is considered affirmative.

The sense in which I am making an affirmative-negative distinction is one that can be extracted from standard predicate logic and is essentially different from that defined by Frege and Ayer (pp. 36-7) among others (Wolfram, p.166). It is based on two types of sentences:

1. Basic singular sentences where the predicate B (simple, as in atomic sentences, or complex, e.g., internal negation, singular belief sentences) is evaluated for its truth value in the base clause of a standard truth condition, e.g., something on the order of 'Ba' is true iff the semantic value of 'a' is a member of the semantic value of 'B', and

2. Negations (external negations) of such singular sentences, i.e., sentences constructed by applying the standard negation sign to such basic sentences. All other sentences treated by standard first order logic can be classified as negative or affirmative depending on whether they are implied by basic negations (negations of such basic singular sentences) in which case they are classified as negative or as requiring for their truth the truth of a basic affirmative sentence and so classified as affirmative.

This is not an arbitrary distinction and a case was made earlier for its utility as a rival account of existential import and ontological commitment. In the present sense of the affirmative-negative
distinction: external negations of basic affirmative sentences are what we mean by basic negations. Internal negations function as components of a predicate and when joined with the appropriate singular terms constitute affirmative sentences.

Frege and Ayer's Two Arguments:

I. The first argument Frege (pp. 380-1) and Ayer (pp. 36-7) offered concerned sentences that are equivalent in English or in some logical notation on the grounds of the purely logical equivalence of their sentence or predicate logic forms. Thus, given the purely formal predicate logic equivalence, of the following:

   'If anything is an A, then it is a B' (which contains no negative expressions)

   and:

   'Nothing is an A that is not a B',

we cannot tell whether to classify 'All A are B' as Affirmative or Negative since it can be represented in terms of either of the preceding sentences. So on Frege's and Ayer's construal of the distinction, one could not decide how to classify 'All A are B'. This type of argument can be duplicated for other sentences.

The affirmative-negative distinction used in this work is arguably closer to the traditional account. On the present distinction, no basic affirmative sentence is equivalent to its negation, and sentences "dependent" on each of these two are not equivalent. Whether 'All A's are B's' is represented in predicate logic as a universal conditional 'If anything is an A, then it is a B' (without any negative expressions) or the negation of a particular generalization 'Nothing is an A that is not a B', on the present distinction it is negative, since it follows from the negation of basic affirmative sentences of the form: $\neg Aa$. Basic negations suffice for its truth, i.e., it is not affirmative since the truth of a basic affirmative is not a necessary condition for its truth. In other words, it would be true with basic negations being true and without any basic affirmative being true. This is precisely what happens with a vacuously true universal generalization of a conditional. When we treat 'All humans are mammals' as equivalent to 'If anything is a human, then it is a mammal', it is negative and has no existential import. It would be true when there are no humans since in that case all sentences of the form 'a is human' are false.

II. The second argument relies on assumptions about non- or extra-logical equivalences. These equivalences involve what Ayer (pp. 60 -1) called complementary predicates; such as 'fat' and 'skinny' (disregard the issue of vagueness), 'blind'/'unsighted' and sighted, or 'blue' and
'not-blue'. In traditional logic this problem arose in connection with what are sometimes called infinite or indefinite terms, e.g., 'infinite' defined as 'non-finite', 'blind' defined as 'non-sighted' or 'having teeth' as 'not-toothless' (Henry, pp. 3-6). In these two pairs, either side is said to be non-logically equivalent to the other. Each is said to be, in some sense of "definable", definable in terms of the other and negation. But just as one might define 'fat' as 'not skinny', so may one define 'skinny' as 'not fat'. On the view that the affirmative-negative distinction depends on the presence of negative expressions, it is arbitrary which of the pairs (and hence the sentences containing them) are negative and which affirmative. My reply will be that this poses a problem for all who do predicate logic and introduce complementary predicates. A scope distinction between external and internal negation provides a solution.

When we apply this problem to our basic affirmative - basic negative distinction, it does seem to be arbitrary whether we introduce 'is fat' as a basic predicate yielding a basic affirmatives or introduce it in terms of 'is not skinny' and so in some sense negative. But this issue raises questions that apply as well to introducing complementary predicates into predicate logic. It is a problem for all. Frege and Ayer were mistaken in thinking that it bears only on those who make much of the quality/quantity distinction and that a solution was not available. The question is one that bears on introducing predicates into logic. Let me first show how it is a problem for all who do predicate logic and wish to allow for the introduction of complementary predicates. When we apply this problem to our basic affirmative - basic negative distinction, it does seem to be arbitrary whether we introduce 'is fat' as a basic predicate yielding a basic affirmatives or introduce it in terms of 'is not skinny' and so in some sense negative. But this issue raises questions that apply as well to introducing complementary predicates into predicate logic. It is a problem for all. Frege and Ayer were mistaken in thinking that it bears only on those who make much of the quality/quantity distinction and that a solution was not available. The question is one that bears on introducing predicates into logic. Let me first show how it is a problem for all who do predicate logic and wish to allow for the introduction of complementary predicates. Let us assume that we have two complementary predicates, e.g., 'is fat' and 'is skinny', and that one is equivalent to the negation of the other (a biconditional obtains between them): a is fat iff a is not skinny. Assuming further that both 'a is fat' and 'a is not skinny' have the same truth value when 'a' is vacuous, e.g., both are false or both are true (as would be required by a uniform account of such vacuous cases), then the biconditional which was assumed to hold does not (since one appears to be the negation of the other).

The solution to this difficulty is to make an external negation - internal negation distinction. It is not an ad hoc decision, since scope distinctions must be made on other grounds as well. When explained as external negation, i.e., 'a is not skinny' is rendered as '¬ (a is skinny). The biconditional does not hold when 'a' is empty. From the standpoint of the present affirmative-negative distinction this means that it is not arbitrary to say which sentences are derived from basic negations (and so do not have existential import) and which do. Assuming that both 'is skinny' and 'is fat' are primitive basic predicates, they both have existential import when forming basic affirmative sentences. Their negations (external) do not. Since such basic predicates are not

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5 This is an adaptation of an argument provided by Elliot Mendelson.
equivalent to the negation (external negation) of their complementary basic predicates, there is not a problem in saying that one and the same sentence has and does not have existential import. With this lack of equivalence there should be no temptation to say that there is one and the same thought-proposition that the two sentences express.

Taking the negation as internal negation, the equivalence holds: a is fat iff a is non-skinny. Both 'is fat' and 'is non-skinny' are basic predicates, though the latter is a complex basic predicate (such complex basic predicates are treated in the base clause of truth conditions). As internal negations, both have existential import and so there is no problem for our account.

The truth condition for such basic internal negation sentences treats the complex predicate 'is non-skinny' as a basic predication, e.g., 'John is non-skinny' is true iff the semantic value of 'John' is a member of the semantic value of 'is non-skinny'. The relation of external to internal negation in such simple contexts is as follows: That \( \neg (a \text{ is skinny}) \) does not imply that a is non-skinny. The former has no existential import while the latter does. That a is non-skinny implies that a exists and that non-skinny things exist and that \( \neg (a \text{ is skinny}) \).

Contrary to what Frege and Ayer thought, no affirmative (in the sense of basic affirmative sentence) is equivalent to a negative one (in the sense of a basic negative sentence).

In summary, Frege and Ayer's first problem does not apply to our distinction. The second problem applies to all who do standard predicate logic and introduce complementary predicate constants; and its solution requires bringing to bear the external-internal negation distinction.
Appendix B: Wittgenstein, Russell and Read

I take it as somewhat uncontroversial that a material adequacy condition for 'all' and 'some' is that they be analogues to 'and' and 'or'. However in lectures and conversations an objection is frequently raised. Russell is cited as though some remarks of his refute this adequacy condition. Russell offered a criticism of Wittgenstein's reduction of universal facts to conjunctive ones. This criticism is interpreted in a rather strong sense, as though Russell showed that conjunction is not connected with universality. But was that Russell's point or was it the more modest claim that in some sense conjunction alone will not do (suffice) for universality? In this appendix I consider ways in which universality and conjunction come apart and ways in which they are connected. My conclusion will be that properly construed Russell’s points do not provide reasons for rejecting the all/some - and/or analogies adequacy condition.

The issue of how strong the relation is between universal generalizations, 'All', and conjunctions, 'and', is quite likely best known in terms of Russell's critical remarks on Wittgenstein's attempted reduction of universality to conjunction (Russell, “The Philosophy of Logical Atomism”, p. 236). Different questions must be asked about the nature of the connection between universal generalizations and conjunctions. Is the connection a priori, analytic, necessary or simply one of material equivalence (merely coextensive)?

Russell's remarks fall into two parts. The first:

I come now to a question which concerns logic more nearly, namely, the reasons for supposing that there are general facts as well as general propositions. ------ I do not think that we can doubt that there are general facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts there are in the world, it is a further-fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. (p. 236)

His point is primarily ontological, concerning an enumeration of facts. He distinguishes different sorts of facts: an enumeration of lower order facts "in the world" and higher order further-facts "about the world". The "in the world" facts are completely enumerated. However, they do not amount to or imply the further fact "about the world", i.e., that these are all the facts (that there are no more facts than those enumerated). The very next sentence in the passage attempts to apply the above to a universal generalization.
The same thing applies to 'All men are mortal'. When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world. (p. 236)

Let us us concentrate on this latter passage and put aside Russell's statement of the issue in terms of an ontology of facts. Instead interpret the passage as though it applied to the relation of a universal generalization (‘All men are mortal’) to a set containing each of the instances of the generalization (‘This human is mortal’, ‘That human is mortal’, etc.). Let us also assume that when you have these instances you can form an exhaustive conjunction made up of them (‘This man is mortal & that man is mortal & etc.’). Questions arise as to the connection between the generalization and the conjunction. Is it in some sense one of a prioricity, of analyticity (and in what sense of "analyticity"), of implication, of co-extensiveness, or is there some element of necessity? The relation must be examined in two directions, that of the universal generalization to the conjunction and that of the conjunction to the universal generalization. This amounts to examining the two conditionals that juxtapose a universal generalization and a suitable conjunction.

The questions can be phrased concerning the relation of a universal generalization to an appropriate conjunction, schematically of (x)(---x---) to (---a--- & ---b--- & etc.), as per whether the biconditional

(x)(---x---) ↔ (---a--- & ---b--- & etc.)

is in some sense apriori, analytic in the sense of having synonymous components, analytic in the sense of being logically true, simply true or containing some element of necessity. The issue is confined here to exhaustive conjunctions. The more interesting of the two conditionals involved is

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6 Exhaustiveness consists of each of the objects in the domain being named and that there is a conjunction of the sentences containing these names.

2. While it is one thing to point out that there are ways of doing logic that do not assume so many names or sentences of infinite length, when dealing here with Russell's argument, we are required to state the problem without ruling out that there could be enough names and sentences. Russell's discussion and his conclusions assume exhaustiveness.

The points made in this appendix may well also, in a sense, apply where initially we do not have exhaustiveness. It seems plausible that wherever exhaustiveness does not obtain, we could embed the framework in one in which there is exhaustiveness. Michael Dummett has proposed a strong connection between generalizations and conjunctions/disjunctions (pp. 516-7). In his interpretation of Frege, Dummett was aware that there may not be enough sentences/names present. He introduced
However for completeness sake consider the less controversial conditional

\[ (x)(\neg \neg x) \rightarrow (\neg \neg a \& \neg \neg b \& \text{etc.}) \] . The relation here is one of logical implication and logical truth given the rules of universal instantiation and those for introducing conjunctions.\(^8\)

Risking misinterpreting Russell, let us take Russell's expressions "newness of a fact" and "inferring it" in an epistemological spirit. We might construe Russell's remarks about the "newness of a fact", as denying that knowledge of a de facto exhaustive conjunction in some way provides an a priori justification of knowledge of the associated universal generalization.\(^9\) Clearly there is a relevant sense in which infinitary truth functions as a device for constructing infinitary conjunctions. He does not elaborate but he may have had in mind conjunctions made up of propositions of some sort and not sentences. One way in which this might be accomplished is with Kaplanesque singular propositions. These are ordered \(n\)-tuples of individuals and properties/relations. Thus for the one-placed monadic sentence 'John is human', we have as the corresponding proposition the ordered pair comprised of the individual John and the property of being human. We can form a conjunction from each and every atomic/singular proposition. Since the objects are a part of the proposition, we don't have a problem of not having sufficient names/propositions. As it stands, this will work only for existing objects. This would not do for the present work. I allow for empty names and for the negative solution (and I do not take a Meinongian stance introducing non-existent objects as parts of such propositions). We might rectify the situation by constructing atomic/singular propositions not out of ordered \(n\)-tuples of individuals and properties/relations, but out of individual properties and properties/relations. So for the sentence 'Vulcan is a planet', we would have as our singular proposition the ordered pair containing the empty property of being identical with Vulcan/Vulcanizes and the property of being a planet. Parallel treatments are available for non-empty names, e.g., the ordered pair comprised of the property of being identical with John and the property of being human. The truth condition for these is similar to the usual one and yields the same negative solution. Such a singular proposition is true iff the value of the singular term, i.e., the property of being Vulcan, applies to an individual which has the property given by the second term of the ordered pair, i.e., being a planet. The Vulcan proposition will be false while the John one would be true. So, with such singular ordered \(n\)-tuple propositions, there appears to be the possibility of using Dummett's thought on constructing the right sort of infinitary truth functions to go with the negative solution adopted in this work and to retain the all/some - and/or analogies.

\(^8\) We can readily defend a weakened form of the all/some - and/or condition. It would rely only on the less controversial one way relations of a universal generalization to a conjunction of its instances and of a disjunction of instances to a particular/existental generalization.

\(^9\) I use the phrase 'de facto exhaustive conjunction' for cases where, as a matter of fact, the conjunction is exhaustive. I represent it as \((\neg \neg a \& \neg \neg b \& \text{etc.})\).

This contrasts with and should not be confused with a logically exhaustive conjunction, \((\neg \neg a \& \neg \neg b \& \text{etc. and there are no other conjuncts of this form})\).
knowing or believing a conjunction (even an exhaustive one) is not tantamount to knowing or believing a generalization. So Russell would be right if he had said that knowledge of (or belief in) an exhaustive conjunction need not suffice for knowledge (or belief) in an associated universal generalization. Russell is also right if we take "inferring" non-epistemologically in the sense of purely first order logical implication. An exhaustive de facto conjunction does not in a purely logical sense imply a corresponding universal claim.

Another way of understanding Russell's point is to construe him as showing that universal sentences are not analytically connected with de facto exhaustive conjunctions. 

\[ \neg ((a \land b \land \text{etc.}) \rightarrow (x)(\neg x)) \] is not analytic. As pointed out in the preceding paragraph the conditional is not a logical truth and so is not analytic in this sense. Neither is it analytic in the sense that the antecedent and the consequent are synonyms. None of these claims were required for the all/some and/or adequacy condition.

So while universal sentences and appropriate de facto conjunctions are not synonymous or logically equivalent, and there is no a priori or purely logical relation between conjunctions and generalizations, this leaves open the question as to whether such biconditionals are true (are coextensive). Stephen Read made an interesting contribution to this topic. The significance of Read's argument consists of showing that such biconditionals are indeed true. I am interpreting his remarks as arguing against Russell and for the truth of the material equivalence, i.e., the biconditional, and, more particularly, for the relevant conditional.

Russell objected that the two propositions [the universal generalization and the de facto exhaustive conjunction], are not equivalent for the second (the long conjunction) needs a final clause [adding that the conjunction is exhaustive], 'and these are all the F's'. I believe he was wrong. If the conjunction was exhaustive (that is contained reference to every F), the two propositions were equivalent; if not, the extra clause is ineffective, since it is false. (Read, 1994, p.48)

As interpreted here, Read's point is that if the conjunction is de facto exhaustive, then the conditional is true, and it is superfluous to its truth to add a component explicitly stating that the conjunction is exhaustive. The bit of jam I would add to his account is a conjecture as to the presence of an element of necessity. This element of

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The newly added conjunct is an exclusion clause. Russell correctly pointed out that with the addition of such an exclusion clause there would be no reduction. The added conjunct involves the very generality that is supposed to be eliminated.

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10 In a personal communication Stephen Read indicated that he wanted something stronger than a material conditional.
necessity comes to light by noting that Read's point applies as well for possible worlds. That is to say, in any possible world where a de facto exhaustive conjunction is true, then so is the appropriate universal generalization. I am not claiming that the conjunction formally implies the generalization. I am calling attention to the same sort of non-strictly logical, non-apriori, non-analytic necessity we find in other areas. Though there are significant differences between water's connection with H2O and suitable exhaustive conjunctions connection with corresponding universal generalizations, neither of these are merely contingent. There is only an illusion of contingency. Given a universe with each of its objects named and the ability to form appropriate de facto exhaustive conjunctions that are true, then the universal generalization is true as well. This is not merely contingently true. Can we conceive of (“could there be”) a possible world with a true exhaustive conjunction without the universal generalization being true as well?

To sum up matters. Russell did not show that universality and conjunction come apart so strongly as to refute the all/some-and/or adequacy condition. On the contrary, there is good reason for maintaining that there is at least a biconditional that holds between the generalization and the conjunction in question. Moreover, given the feasibility of extending any non-exhaustive conjunction to an exhaustive one, a similar biconditional can hold for other cases as well.

There are several ways in which the connection between the respective quantifiers and the connectives has been and can be recognized. The connection provides a rationale for the pedagogical technique of introducing quantification theory via conjunctive and disjunctive expansions. It is given recognition notationally in the California (Kalish and Montague) style of writing the quantifier signs as larger (Infinite) conjunction and disjunction signs. Since sums and products are related to disjunctions and conjunctions, Peirce’s sum/product notation for the quantifiers has something in common with the California disjunction/conjunction notation. (Bochenski, p. 349) The connection is the basis of Behmann’s decision procedure for monadic predicate logic. The connection was part of the doctrine of descent to the singular of Terminist logic. Ockham, Buridan, etc. where categorical sentences were dealt with in terms of conjunctions and disjunctions. But these forms of recognition don’t go far enough. The nature of this relation suggests that accounts in first order predicate logic and the philosophy of logic which do not abide by the connection, are likely to misconstrue an important feature of the quantifiers. How should one take cognizance of this connection? Two proposals come to mind:

1. Take the connection as providing the basis for giving truth conditions for generalizations, i.e., give truth conditions for the
quantifiers in terms of the associated conjunctions and disjunctions. (See the formalization of the doctrine of descent to singulars of supposition theory in G. Priest and S. Read, "Merely Confused Supposition" p. 268.)

2. Take the connection as providing some form of material adequacy condition for accounts of the quantifiers. The latter, more modest course, is the one taken in this work. Having said that, all of the main points made in terms of these analogies can be made without them, by appealing solely to the standard rules of predicate logic and the Mates-like truth conditions.